A SIMPLE MODEL OF THE PLASMA DEFLAGRATION GUN INCLUDING SELF-CONSISTENT ELECTRIC AND MAGNETIC FIELDS

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Introduction

At the Air Force Weapons Laboratory, interest has continued for some time in energetic plasma injectors. A possible scheme for such a device is the plasma deflagration gun. When the question arose whether it would be possible to scale a deflagration gun to the multi-megajoule energy level, it became clear that a scaling law which described the gun as a circuit element and allowed one to confidently scale gun paramenters would be required. We sought to develop a scaling law which self-consistently described the current, magnetic field, and velocity profiles in the gun. We based this scaling law on plasma parameters exclusively, abandoning the fluid approach used by Cheng.[1-4]

Scaling Law

Our one-dimensional quasi-static plasma deflagration gun is shown in Figure 1. The boundries at $y = 0$ and at $y = D$ are perfect conductors. The boundry at $y = 0$ is maintained at a constant voltage V with respect to the opposite boundry so that a constant electric field E_y exists throughout the volume of the gun. The flow of current through the yet unspecified plasma within the volume of the gun yields a current density $J_y(z)$ and a magnetic field $B_x(z)$ within the volume. The current path enters at $y = 0$, $z = z_b$, $(z_b > 0)$, crosses between the electrodes throughout the volume of the gun, and exits at $y = D$, $z = z_b$ so that the magnetic field is zero at the muzzle of the gun, $z = z_m$, $(z_m < 0)$ and the plasma moves in the $-z$ direction.

We begin by assuming a low density flow, dominated by single particle dynamics in which plasma motion is driven by the $\vec{E} \times \vec{B}$ drift of the particles as they orbit the magnetic field lines. The bulk plasma velocity is given by

$$v = \frac{\vec{E} \times \vec{B}}{B^2} = \frac{E}{B}(-\hat{z}) \tag{1}$$

Now, unless the current distribution extends to $z = -\infty$ (which would require infinite magnetic field energy) $B_x = 0$ at some station of the gun. Further, we could simply extend the conductor such that $E_y = E$ where $B_x = 0$ and this mechanism would imply an infinite velocity for the plasma. Since this is unphysical, v must plateau at some value $v = v_{final}$. A possible explanation of this cutoff in velocity is the particles' Larmor radius exceeding the interelectrode spacing $(r_L > D/2)$, but for purpose of this discussion, it is sufficient to say that some mechanism does exist.

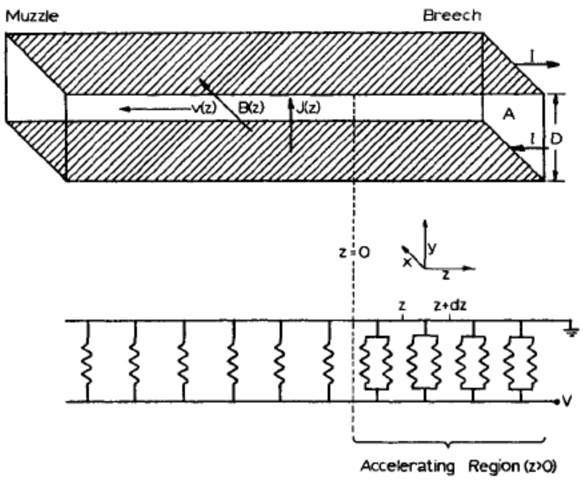

Figure 1. 1-dimensional quasi-static plasma deflagration gun.

We set $z = 0$ at the point where acceleration ceases and where $v = v_{final}$. We identify the magnetic field at that point as $B_x = B_o$, so that

$$v_{final} = \frac{E}{B_o} \tag{2}$$

From $z = 0$ to the muzzle of the gun at $z = z_m$, the only current flowing is the conduction current $\vec{J} = \sigma_{th}\vec{E}$ resulting from the fact that the plasma has some finite temperature and hence some finite conductivity σ_{th}. We assume that the plasma temperature and conductivity are constant for all z (including $z > 0$) so that $\vec{J}_c$ is constant. For completeness we assume that the power input into the volume by $\vec{J}_c \times \vec{E}$ is balanced through radiative and conductive losses. This is necessary if we are to find a steady-state solution to the problem, and is consistent with the constant temperature assumption.

If particles enter the gun at positive z and are accelerated until they reach the cutoff point, there must be an additional input of electrical power into the volume for $z < 0$. We allow this additional power, input through the presence of a current density J_d such that $\vec{J}_d \times \vec{E}$ contributes only to the drift of the particles, and it is this current density for which we wish to find a self-consistent solution. We begin by describing the dependence of $B(z)$ on $J(z) = J_c + J_d(z)$. We will solve the problem quasi-statically and one-dimensionally; that is, we will allow variations on the z-direction only,

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = \frac{\partial}{\partial t} = 0 \tag{3}$$

It is easy to see from Ampere's law that for $z > 0$,

$$B(z) = B_o + \mu_o J_c Z + \mu_o \int_0^z J_d(z')dz' \tag{4}$$

so that we may write

$$\frac{dB(z)}{dz} = \mu_o J_c + \mu_o J_d(z) \tag{5}$$

Now, if we say that the bulk plasma velocity is simply the $\vec{E} \times \vec{B}$ drift velocity at any station in the gun, we have

$$v^2(z) = \frac{E^2}{B^2(z)} \tag{6}$$

and

$$\frac{dv^2(z)}{dz} = -2\frac{E^2}{B^3(z)}\frac{dB(z)}{dz} \tag{7}$$

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **JUN 1985**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**
4. TITLE AND SUBTITLE **A Simple Model Of The Plasma Deflagration Gun Including Self-Consistent Electric And Magnetic Fields**		5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Air Force Weapons Laboratory Kirtland AFB, NM 87117-6008**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release, distribution unlimited**		
13. SUPPLEMENTARY NOTES **See also ADM002371. 2013 IEEE Pulsed Power Conference, Digest of Technical Papers 1976-2013, and Abstracts of the 2013 IEEE International Conference on Plasma Science. Held in San Francisco, CA on 16-21 June 2013. U.S. Government or Federal Purpose Rights License.**		
14. ABSTRACT		
15. SUBJECT TERMS		

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a REPORT **unclassified**	b ABSTRACT **unclassified**	c THIS PAGE **unclassified**	**SAR**	**4**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

We choose to evaluate v^2 because it is related to the directed kinetic energy of the ions, by which the bulk of the momentum and energy are carried. Per particle,

$$K(z) = \left(\frac{m_i}{2}\right) v^2(z) \tag{8}$$

$$\frac{dK(z)}{dz} = \left(\frac{m_i}{2}\right) \frac{dv^2(z)}{dz} \tag{9}$$

Now, if the particles are streaming through the gun so that the particle flux is Φ, the differential electrical power per unit length input into bulk plasma motion (recalling that particle energy is increasing with decreasing z) is

$$\frac{dP(z)}{dz} = -E\, J_d(z) A \tag{10}$$

But the differential power input must be related to particle kinetic energy by

$$\Phi A \frac{dK(z)}{dz} = \Phi A \left(\frac{m_i}{2}\right) \frac{dv^2(z)}{dz} \tag{11}$$

$$= \Phi A \left(\frac{m_i}{2}\right) (-2)\frac{E^2}{B^3(z)} \frac{dB(z)}{dz} \tag{12}$$

$$= \Phi A m_i E^2 \left[B_o + \mu_o J_c z + \mu_o \int_0^z J_d(z')dz'\right]^{-3}$$

$$\cdot \left[\mu_o J_c + \mu_o J_d(z)\right] \tag{13}$$

Equating the two expressions yields

$$-E\, J_d(z)A = -\Phi A m_i E^2 \left[B_o + \mu_o J_c z + \mu_o \int_0^z J_d(z')dz'\right]^{-3}$$

$$\cdot \left[\mu_o J_c + \mu_o J_d(z)\right] \tag{14}$$

Solving for J (z), we arrive at the integral equation

$$J_d(z) = J_c \frac{m_i \Phi E \mu_o}{\left[B_o + \mu_o J_c z + \mu_o \int_0^z J_d(z')dz'\right]^3 - m_i \Phi E \mu_o} \tag{15}$$

This equation may be evaluated numerically to arrive at a current profile, but we may readily solve for J at z = 0 where acceleration ceases.

$$J_d(0) = J_c \frac{m_i \Phi E \mu_o}{B_o^3 - m_i \Phi E \mu_o} \tag{16}$$

Note at $J_d(0)$ becomes quite small for small $m_i \Phi E \mu_o$, and becomes negative (corresponding to energy extracted from the plasma, an unphysical solution) for $m_i \Phi E \mu_o > B_o^3$. Therefore, we must operate the gun such that

$$m_i \Phi E \mu_o \leq B_o^3 \tag{17}$$

Recalling that $v_{final} = E/B_o$, we rearrange terms to get

$$m_i \Phi v_{final} \leq \frac{B_o^2}{\mu_o} \tag{18}$$

In other words, the pressure (or momentum flux) of the plasma stream cannot exceed twice the magnetic pressure at cutoff. It is noteworthy that this is similar (within a factor of two) to the result which Cheng derrived by eliminating terms of the momentum equation by estimating orders of magnitude.[2] We may define an effective beta

$$\beta_{eff} = \frac{m_i \Phi v_{final}}{(B_o^2/2\mu_o)} \tag{19}$$

For a deflagration gun operating with maximum output of momentum flux (maximum effective beta), $J_d(0)$ is infinite; that is, power input into the bulk plasma motion takes place in an infinitely thin current sheath. For finite $J_d(0)$, the current sheet has finite thickness, but an examination of equation (18) shows that the larger $J_d(0)$, the quicker $J_d(z)$ falls off with increasing z. In short, our simple model returns a result that has been verified in experiment: the current distribution in a deflagration gun is composed of a diffuse current throughout the barrel and a thin current sheath of high current density at the breech. (See Figure 2.)

<u>Comparison with Experiment</u>

Let us consider the B-field profile that would be observed in a deflagration gun operating at maximum efficiency ($\beta_{eff} = 2$), accelerating particles from rest to $v = v_{final}$ in a infinitely thin jump region. Granted, this limit is unattainable, but it is the simplest to analyze. We integrate over the delta-function current profile J_d to obtain the current in the jump region I_d. Power balance requires that

$$E\, DI_d = \left(\frac{m_i}{2}\right) v_{final}^2 \,\Phi\, A \tag{20}$$

Now, we may use our two equivalent expressions for v_{final}, equation (2) and a derivation from equation (19),

$$v_{final} = \beta_{eff} \left(\frac{B_o^2}{2\mu_o m_i \Phi}\right) \tag{21}$$

along with Ampere's law,

$$\Delta B = \frac{\mu_o I_d D}{A} \tag{22}$$

to solve for the step increase ΔB across the current sheath. Combining equations (2),(20),(21),and (22) yields the result

$$\Delta B = \frac{\beta_{eff}}{4} B \tag{23}$$

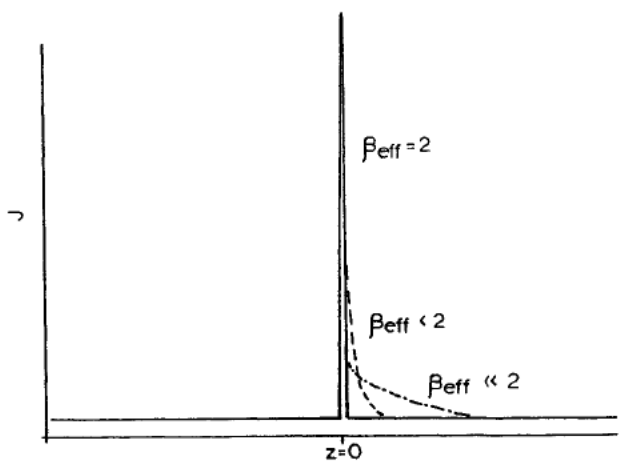

Figure 2. Current density in the model gun.

This analysis gives favorable comparison with experiment. Figure 3 shows a magnetic field contour measured by Cheng during a quasi-steady discharge phase in a coaxial plasma deflagration gun.[1] Figure 4 shows the current density on the outer electrode derived from this magnetic field profile. Overlaid on these plots is the magnetic field profile predicted by our model if the current density J_d were a delta function at the peak of the calculated current distribution and J_c were sufficient to support the magnetic field in the remainder of the gun. Certainly the agreement is not perfect, but it is qualitatively correct, and is a good fit considering we are comparing an idealized, rectilinear model with an actual, coaxial experiment.

Another result falls out of this analysis of the maximum-efficiency gun. If the integrated current density J_d yields a current I_d, and if the jump in the magnetic field is $\Delta B = B_o/2$, then it is clear that

$$I_d = \frac{I_{net}}{3} \qquad (24)$$

Thus, for a constant electric field E , only one-third of the power input into our maximum-efficiency gun goes into acceleration of particles, while two-thirds is dissipated. Hence, measurements which claim that efficiencies of 70 percent or greater are possible with the deflagration gun are highly suspect.

Suggestions for Further Work

It would be highly desirable to build deflagration guns in the megajoule energy range and beyond and at the gigawatt power level. However, deflagration guns which have been built to date occupy a small area of parameter space. Table 1 lists some of the parameters of deflagration guns which have been built by various experimenters and for which a body of data is available. Also listed is a fictitious "typical" deflagration gun. Note that the parameters of the guns which have been constructed vary from those of the typical gun by thirty percent at the most, often by a much smaller margin.

If we ask which parameters we might change to build a more powerful deflagration gun, the simple answer is found by treating the gun as a resistive element, specifically by a pair of parallel resistors R_d and $R_c = R_d/2$. If the gun is filled with plasma of resistivity ρ_{th}, then we simply have

$$R = \rho_{th} \frac{D}{LW} \qquad (25)$$

For a given voltage source, we increase the power dissipation of our resistive element by lowering its resistance. Looking at equation (24), we probably can have little effect on the plasma resistivity. So, we make the gun thinner, or longer, or both (increasing its aspect ratio), or larger in diameter. The most attractive, from the standpoint of beam collimation, is increasing the aspect ratio. This simple analysis indicates that high-aspect-ratio deflagration guns should dissipate higher power than those that have been constructed to date.

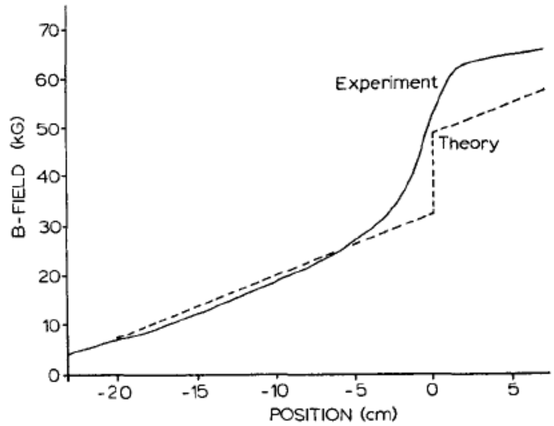

Figure 3. Magnetic field distribution in the gun -- theory versus experiment.

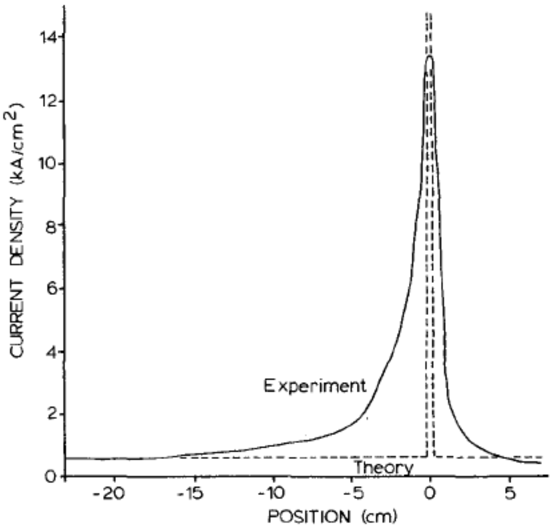

Figure 4. Current density distribution in the gun -- theory versus experiment.

One would feel more comfortable about designing an experiment if a more sophisticated analysis of the problem could be conducted. Analysis of the deflagration gun problem using the one-dimensional code MAGPIE is planned at AFWL. Figure 5 illustrates the initial conditions for this problem. Orginally designed to analyze an imploding plasma liner in a cylindrcal geometry, by setting the radius to a large value and scaling components of the driving circuit apropriately, it is possible to do the rectilinear deflagration gun problem.

Experimenter or Group	Charge Voltage	Maximum Current	Inner Radius	Outer Radius	Aspect Ratio	Gas Species	Output Velocity	Output Density
D. Y. Cheng[1-2]	20 kV	0.65 MA	0.63 cm	4.5 cm	7.6	He	3 - 12 cm/μs	10^{15} - 10^{16} cm
Case Western[6]	9 kV	-	0.50 cm	4.0 cm	9.1	He	10 - 12 cm/μs	10^{15} - 10^{16} cm
AFWL[5]	18 kV	0.42 MA	0.65 cm	2.5 cm	12.3	H,He,Ar	11 cm/μs	3-5 x 10^{15} cm
Typical	20 kV	0.50 MA	0.60 cm	4.0 cm	10.0	He	11 cm/μs	5 x 10^{15} cm

Table 1. Comparison of parameters of several plasma deflagration guns.

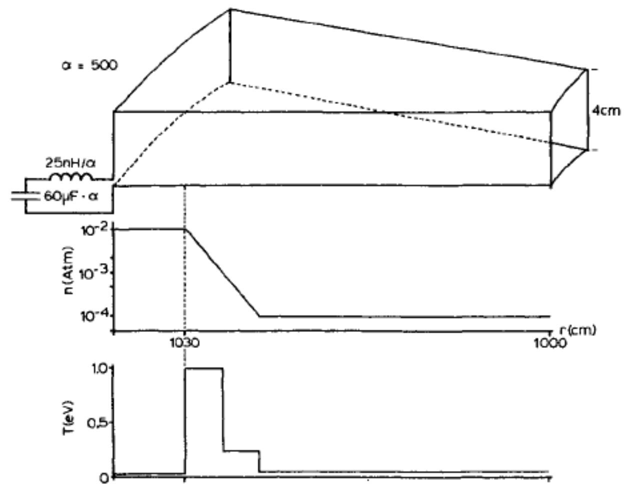

Figure 5. MAGPIE 1-dimensional problem.

Conclusion

We have developed a simple model of the plasma deflagration gun which includes a self-consistent description of magnetic field and current density within the volume of the gun. The results of this analysis are in agreement with data available from deflagration gun experiments. The analysis points to high-aspect ratio designs for deflagration gun experiments in high-energy, high-current regimes.

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